Upstream swimming and Taylor dispersion of active Brownian particles

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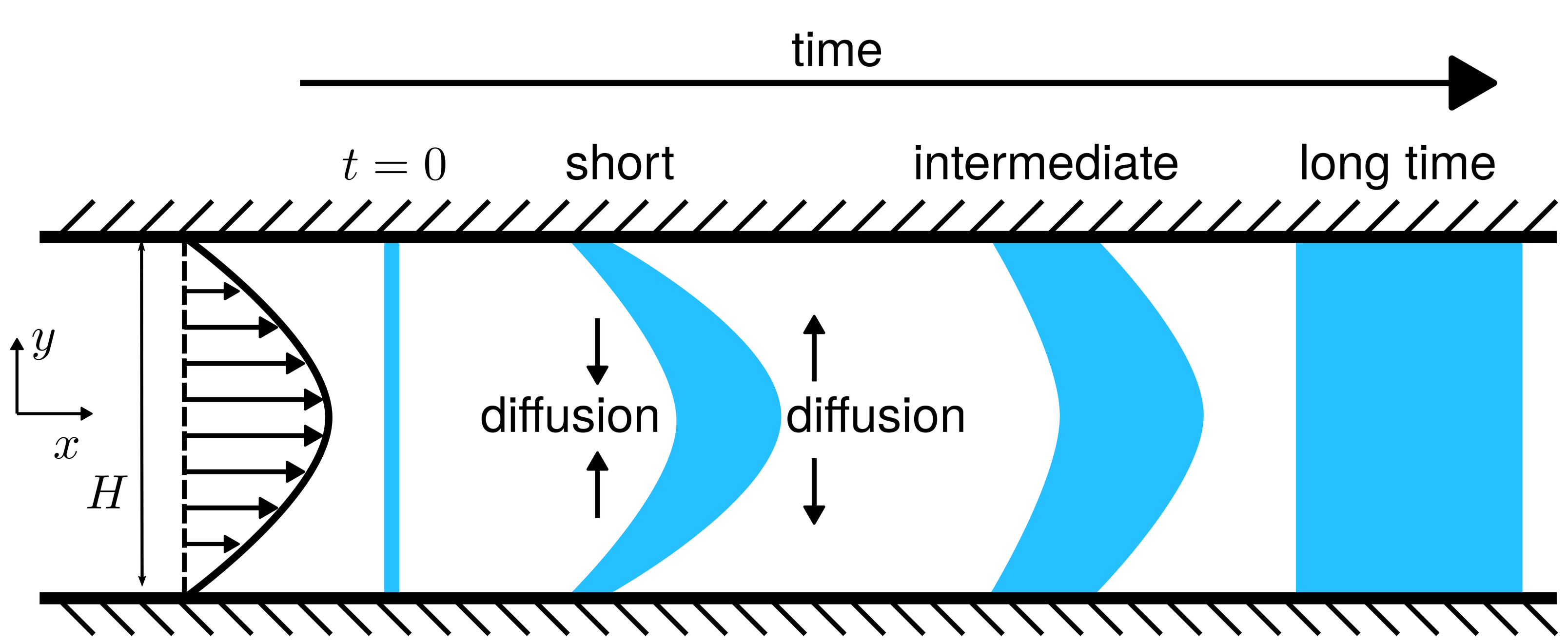
Introduction

The transport of a solute that experiences molecular diffusion and fluid advection in a Poiseuille flow has been extensively studied. In the long time limit, the cross-sectional average number density $\overline{n}$ satisfies an effective advection-diffusion equation,

$$\frac{\partial \overline{n}}{\partial t} + U^{\mathrm{eff}} \frac{\partial \overline{n}}{\partial x} = D^{\mathrm{eff}} \frac{\partial^2 \overline{n}}{\partial x^2}.$$

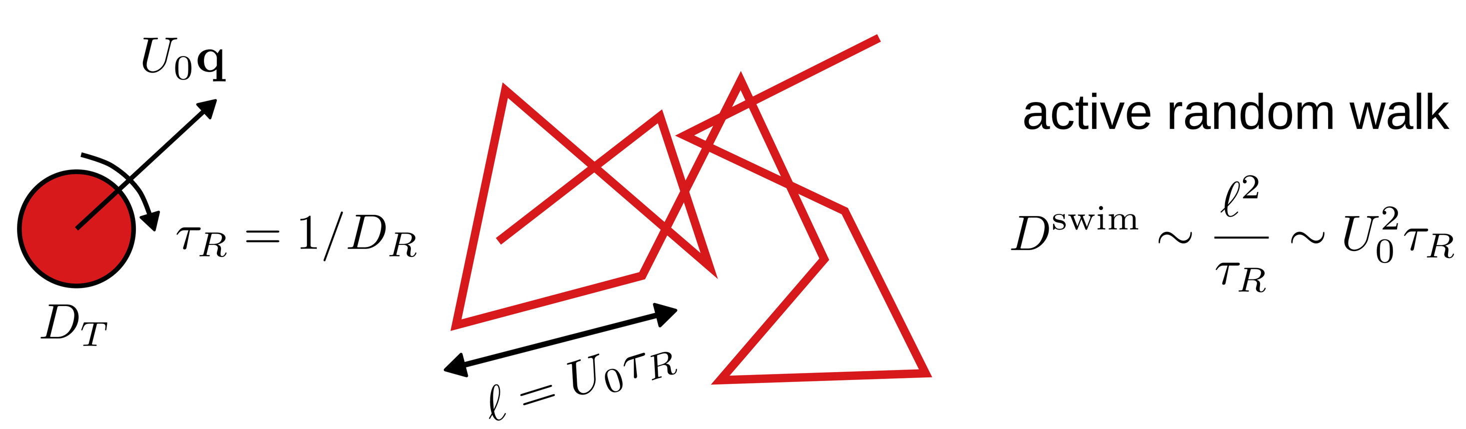


Number density (or concentration) of a Brownian tracer/solute

The average drift is the average flow speed, $U^{\mathrm{eff}} = \overline{u}$. The effective dispersion coefficient (D^{eff}) of the solute is enhanced compared to its bare molecular diffusivity. This phenomenon of shear-enhanced longitudinal spreading is known as Taylor dispersion [1].

Active Brownian particles

An acitve Brownian particle (ABP) self-propels with a fixed intrinsic speed U_0 and undergoes translational and rotational Brownian motion with diffusivities D_T and D_R. The body-fixed unit vector $\mathbf{q}$ denotes the direction of swimming.



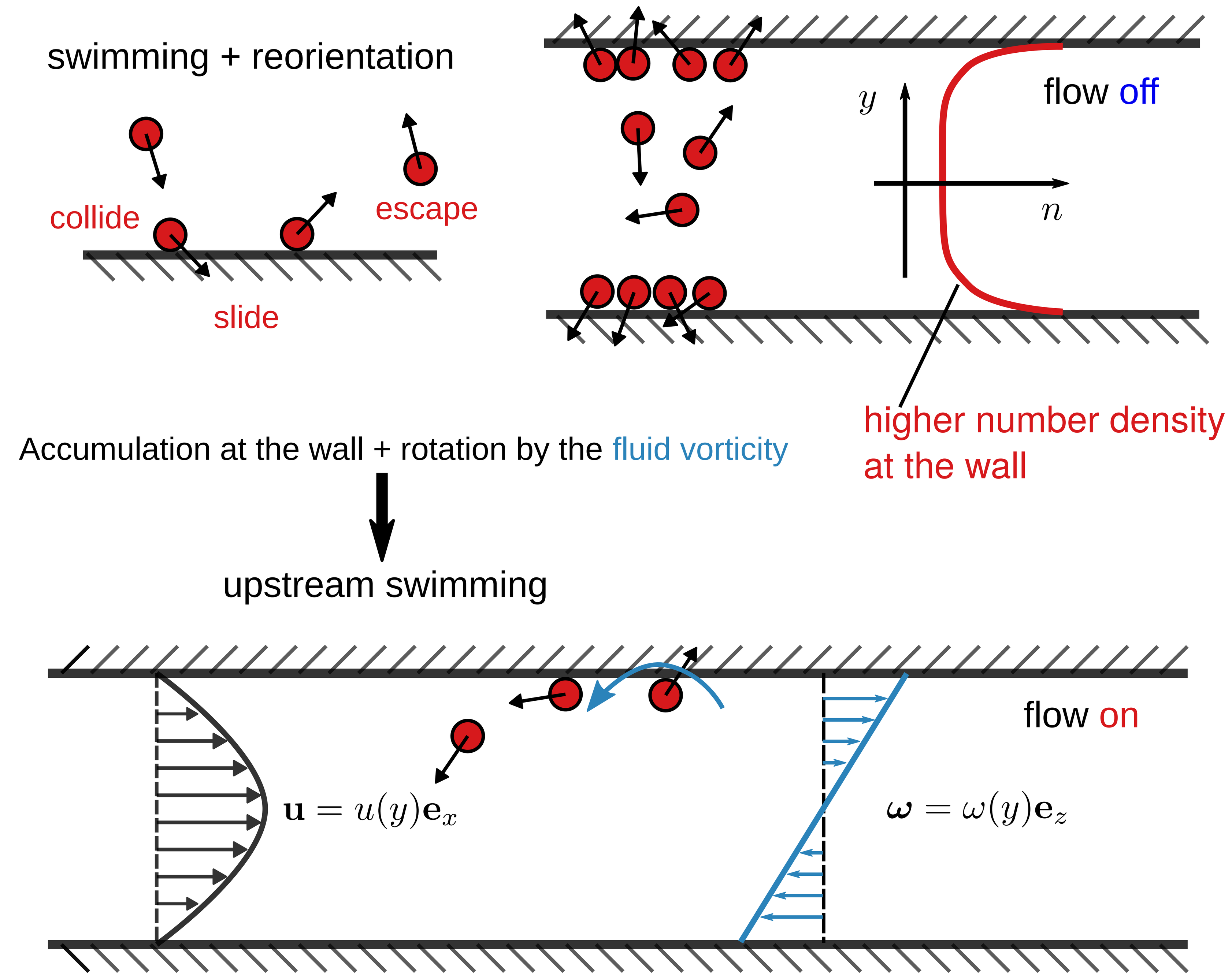
At long times, the swimming motion becomes a random walk due to Brownian reorientation [2-3]. The diffusivity associated with this random walk is called the swim diffusivity.

ABPs in Poiseuille flow

It has been shown that microswimmers in Poiseuille flow exhibit interesting behavior including upstream swimming and non-monotonic dispersion as a function of the flow speed [4-6].

Why do microswimmers swim upstream?
What is the mechanism of non-monotonic dispersion?
Can we explain these phenomena with the simple ABP model?

ABPs accumulate at confining boundaries because they self-propel and cannot penerate the wall [7].



Theoretical framework

The evolution of the single-particle probability density function $P(\mathbf{x}, \mathbf{q}, t)$ satisfies the Smoluchowski equation.

$$\frac{\partial P}{\partial t} + \nabla \cdot \mathbf{j}^T + \nabla_R \cdot \mathbf{j}^R = 0$$

translation

$$\mathbf{j}^T = U_0 \mathbf{q} P + \mathbf{u} P - D_T \nabla P$$

swimming — fluid advection — diffusion

rotation

$$\mathbf{j}^R = \frac{1}{2} \nabla \times \mathbf{u} P - D_R \nabla_R P$$

fluid vorticity — diffusion

Non-monotonic dispersion?

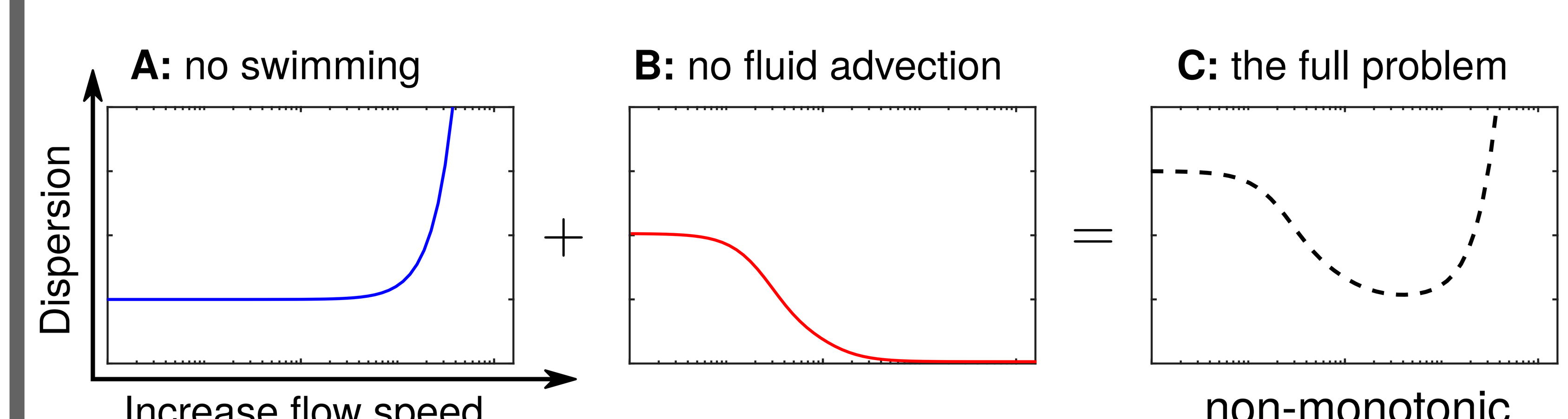
To understand the origin of non-monotonicity, we consider two separate problems, **A** and **B**.

A: $U_0 = 0$, classical Taylor dispersion is recovered.

$$\left(D^{\mathrm{eff}} - D_T\right)/D_T \sim Pe^2 \qquad Pe = u_0 H / D_T$$

B: The fluid velocity $\mathbf{u} = \mathbf{0}$ but the vorticity is retained. No classical Taylor dispersion. Rotation by vorticity reduces the swim diffusivity.

$$\ell^{\mathrm{eff}} \to 0 \text{ as } Pe \to \infty.$$



Results

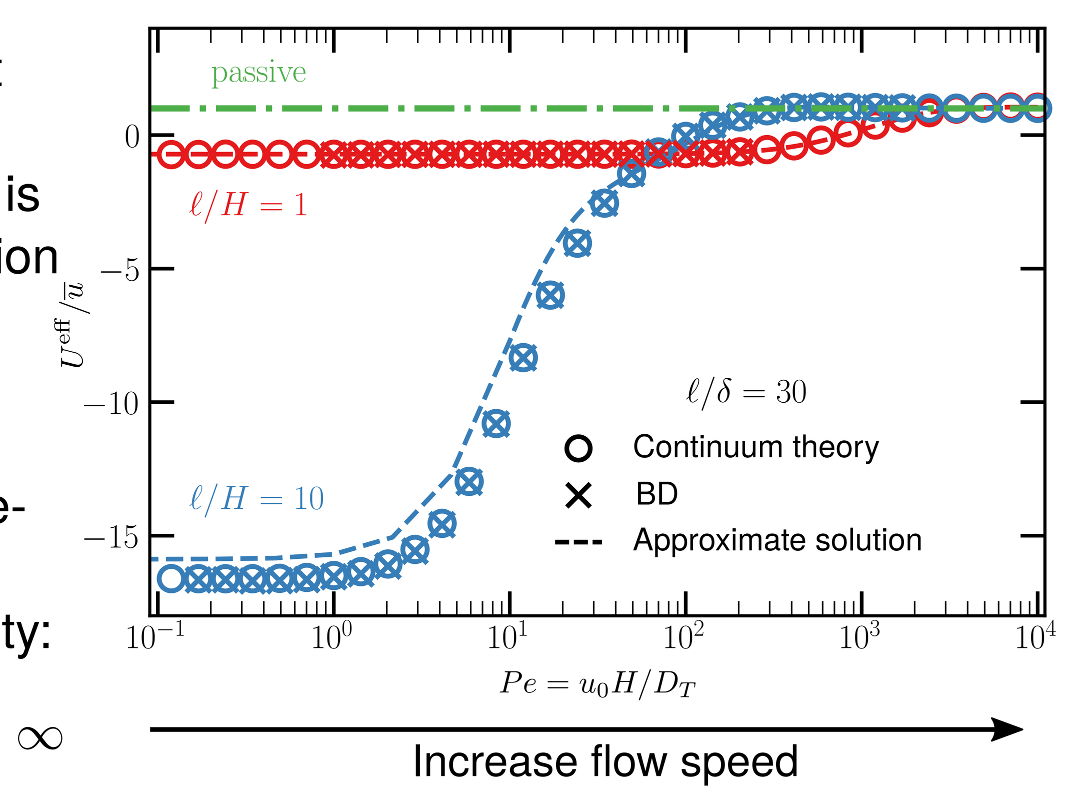
The average drift and effective longitudinal dispersion as a function of the flow speed for different levels of confinement.

$$\ell/\delta = \ell/\sqrt{D_T \tau_R} = 30, \quad \text{high activity}$$

A transition from net upstream motion to downstream motion is observed as a function of the flow speed.

ABPs lose their persistence (passive-like) due to rapid spinning from vorticity:

$$U^{\mathrm{eff}}/\overline{u} \to 1 \text{ as } Pe \to \infty$$

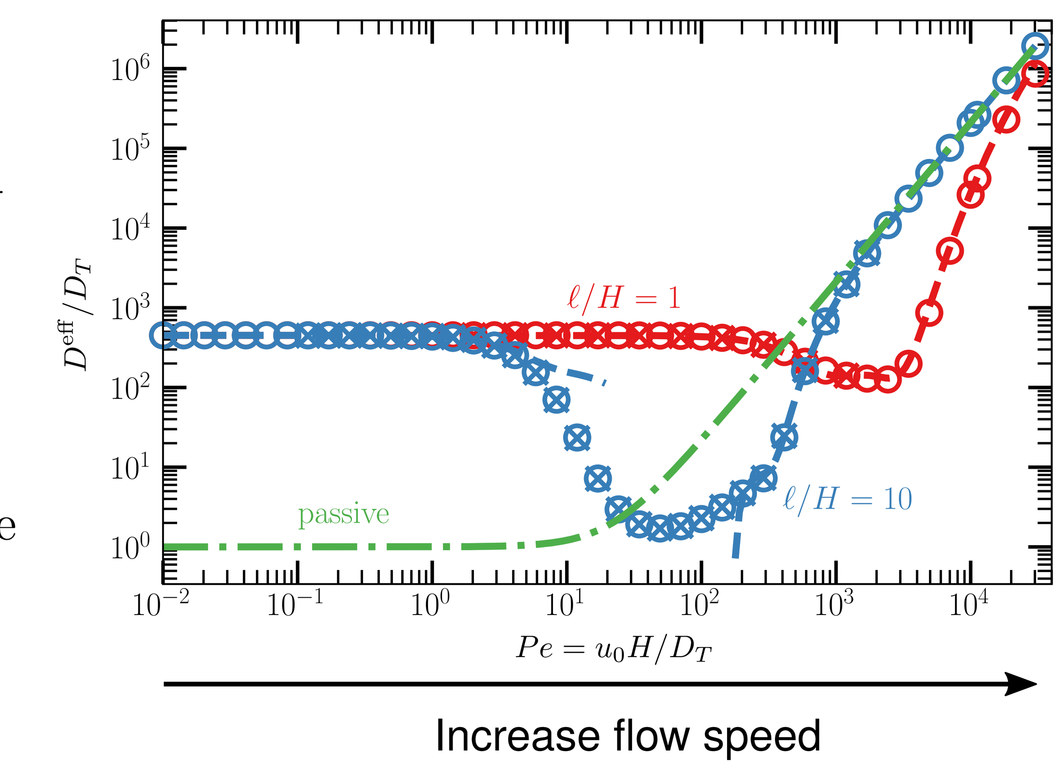


$$Pe = 0$$

$$\frac{D^{\mathrm{eff}}}{D_T} = 1 + \frac{D^{\mathrm{swim}}}{D_T}$$

$$\frac{D^{\mathrm{swim}}}{D_T} = \frac{1}{2}\left(\frac{\ell}{\delta}\right)^2$$

$$Pe \to \infty: \quad \text{passive}$$



Conclusion

Using the generalized Taylor dispersion theory [8], we derived an effective advection-diffusion equation for the cross-sectional average of the particle number density. We have shown that rotation by vorticity provides a robust mechanism for upstream swimming regardless of the type of microswimmer. The combination of flow and activity leads to a non-monotonic variation of the effective longitudinal dispersion as a function of the flow speed.

References

[1] Taylor, *Proc. R. Soc. A*, **219**, 186-203, (1953)
[2] Berg, *Random walks in biology*, (1993)
[3] Takatori & Brady, *Phys. Rev. Lett.*, **113**, 028103, (2014)
[4] Hill *et. al*, *Phys. Rev. Lett.*, **98**, 068101, (2007)
[5] Kaya & Koser, *Biophys. J.*, **102**, 1514-1523, (2012)
[6] Chilukuri *et. al*, *Phys. Fluids*, **27**, 031902, (2015)
[7] Yan & Brady, *J. Fluid Mech.*, **785**, R1, (2015)
[8] Frankel & Brenner, *J. Fluid Mech.*, **204**, 97-119, (1989)